

Magnetic moments of octet baryons at finite density and temperature

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Abstract

We investigate the change of magnetic moments of octet baryons in nuclear matter at a finite density and temperature. Quark-meson coupling models are employed in describing properties of octet baryons and their interactions. Magnetic moments of octet baryons are found to increase non-negligibly as density and temperature increase, and we find that temperature dependence can be strongly correlated with the quark-hadron phase transition. Model dependence is also examined by comparing the results from the quark-meson coupling (QMC) model to those by the modified QMC (MQMC) model where the bag constant is assumed to depend on density. Both models predict sizable dependence on density and temperature, but the MQMC model shows a more drastic change of magnetic moments. Feasible changes of the nucleon mass by strong magnetic fields are also reported in the given models.

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I. INTRODUCTION

Recently, the magnetic moment of a Λ in ${}^7_\Lambda\text{Li}$ was observed in BNL to test its medium modification. It shows us that the magnetic moment may be changed in nuclear medium although the error is still very large [1]. The experiment might be able to extend to the heavy ion collision which can make hot and dense nuclear matter. In this context, it would be interesting to study change of magnetic moments of baryons in hot and dense matter. The investigation for the change of baryon properties in hot and dense matter is important in the interpretation of many exotic phenomena occurring in the proto-neutron star and the heavy ion collision. The subject has been studied by various models such as relativistic mean fields (RMF) models [2, 3], chiral motivated models [4, 5], and so on. In specific, the quark-meson coupling (QMC) model [6], one of the RMF models, is found to effectively describe exotic nuclear matter as well as finite nuclei.

The QMC model is one of the extension of quantum hadrodynamics (QHD) in which interactions between baryons are mediated by the exchange of σ and ω mesons, describing the attraction and the repulsion, respectively. But, in the QMC model, quarks inside baryons interact directly with meson fields. One of merits is that one can evaluate properties of baryons with quark degrees of freedom. For instance, effective masses of baryons in nuclear medium can be obtained from the calculation of the MIT bag by considering quark energies. Likewise, magnetic moments of baryons can be also calculated with SU(6) quark wave functions and a bag radius as well. In the present work, we investigate effective masses and magnetic moments of baryon octet in hot and dense matter.

Under the assumption that the matter reaches to thermal equilibrium, the matter can be described through the minimum of a thermal grand potential or the maximum of the pressure. Applying the maximum condition of the pressure at finite temperature, σ and ω mesons can be self-consistently determined, giving rise to the change of magnetic moments of baryons. Our results may be relevant to the QCD phase transition and medium effect in high-energy nuclear collisions.

The paper is organized as follows. In Sec. II, the QMC model for hot and dense matter is briefly explained. Magnetic moments of baryons are obtained from SU(6) quark wave functions. Results and discussions are followed in Sec. III. Sec. IV is devoted to the summary.

II. MODEL

Since one can find details of the description of hot and dense nuclear matter with the QMC model in Ref. [7], in this section, we address the most essential ingredients of the model. If we ignore the excitation of quarks in a baryon, the QMC model treat a nucleon as a MIT bag in hot and dense nuclear matter. The quark field ψ_q inside the bag satisfies the Dirac equation

$$[i\gamma \cdot \partial - (m_q - g_\sigma^q \sigma) - g_\omega^q \gamma^0 \omega_0] \psi_q = 0, \quad (1)$$

where m_q ($q = u, d, s$) is the bare quark mass, σ and ω_0 are the mean fields of the σ and ω mesons, respectively, and g_σ^q and g_ω^q are coupling constants between quarks fields and the meson fields. We assume $m_u = m_d = 0$ and $m_s = 150$ MeV for bare quark masses.

The ground state solution of the Dirac equation is given by

$$\psi_q(\mathbf{r}, t) = \mathcal{N}_q \exp(-i\epsilon_q t/R) \begin{pmatrix} j_0(x_q r/R) \\ i \beta_q \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} j_1(x_q r/R) \end{pmatrix} \frac{\chi_q}{\sqrt{4\pi}}, \quad (2)$$

with

$$\mathcal{N}_q^{-2} = 2R^3 j_0^2(x_q) [\Omega_q(\Omega_q - 1) + R m_q^*/2]/x_q^2, \quad (3)$$

$$\epsilon_q = \Omega_q + g_\omega^q \omega_0 R, \quad (4)$$

$$\beta_q = \sqrt{\frac{\Omega_q - R m_q^*}{\Omega_q + R m_q^*}}, \quad (5)$$

$$\Omega_q = \sqrt{x_q^2 + (R m_q^*)^2}, \quad (6)$$

$$m_q^* = m_q - g_\sigma^q \sigma, \quad (7)$$

where R is the bag radius. $j_0(x)$ and $j_1(x)$ are the spherical Bessel functions, and χ_q is the quark spinor. The value of x_q is determined from the boundary condition on the bag surface

$$j_0(x_q) = \beta_q j_1(x_q). \quad (8)$$

The energy of a baryon with ground state quarks is given by

$$E_b = \sum_q \frac{\Omega_q}{R_b} - \frac{Z_b}{R_b} + \frac{4\pi}{3} R_b^3 B_b, \quad (9)$$

where B_b is the bag constant, and Z_b is a phenomenological constant introduced to take into account the zero-point motion of the baryon. Subscript ‘ b ’ denotes species of a baryon. In

the QMC model, the bag constant B_b is independent of density and temperature, whereas in the modified QMC (MQMC) model it is assumed to depend on density and temperature. In this work, we employ the direct coupling form given in Ref. [8],

$$B_b(\sigma) = B_{b0} \exp \left(-\frac{4g_\sigma^b \sigma}{m_N} \right). \quad (10)$$

As shown later on, σ -field is a function of density and temperature, so that the bag constant B_b depends on density and temperature in the MQMC model. Effective mass of a baryon b in hot and dense matter is given by

$$m_b^* = \sqrt{E_b^2 - \sum_q \left(\frac{x_q}{R_b} \right)^2}. \quad (11)$$

The value of bag radius in free space is usually chosen close to the radius of the nucleon charge form factor. In this work, we choose $R_b = 1.0$ fm for free-space bag radius. We determine the remaining parameters B_{b0} and Z_b to reproduce the empirical value of the baryon mass in free space with the minimization condition

$$\frac{\partial m_b^*}{\partial R_b} = 0. \quad (12)$$

Numerical values of B_{b0} and Z_b for each baryon can be found in Ref. [10]. Quark-meson coupling constants g_σ^q , g_ω^q and g_σ^b are fitted to reproduce the binding energy per a baryon in infinite nuclear matter (16 MeV) and reasonable compression modulus (~ 280 MeV) at the saturation density ($\rho_0 = 0.17 \text{ fm}^{-3}$) under zero temperature. Numerical values of the coupling constants are given in Ref. [9].

In hot matter, pairs of nucleon and antinucleon are produced and thus the energy density is calculated as

$$\varepsilon = \sum_b \frac{\gamma}{(2\pi)^3} \int d^3k \sqrt{k^2 + m_b^{*2}} (f_b + \bar{f}_b) + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\sigma^2 \sigma^2, \quad (13)$$

where γ is the spin-isospin degeneracy factor. Functions f_b and \bar{f}_b are the Fermi-Dirac distributions for baryons and antibaryons

$$f_b = \frac{1}{e^{(\epsilon_b^* - \mu_b^*)/T} + 1}, \quad (14)$$

$$\bar{f}_b = \frac{1}{e^{(\epsilon_b^* + \mu_b^*)/T} + 1}, \quad (15)$$

where $\epsilon_b^* = \sqrt{k^2 + m_b^{*2}}$ is the effective energy and $\mu_b^* = \mu_b - g_{\omega b}\omega_0$ is the effective chemical potential. If a baryon density ρ_b is given, we can determine the chemical potential μ_b from

$$\rho_b = \frac{\gamma}{(2\pi)^3} \int d^3k (f_b - \bar{f}_b), \quad (16)$$

where ω_0 -meson field in f_b and \bar{f}_b is determined by

$$\omega_0 = \sum_b \frac{g_{\omega b}}{m_\omega^2} \rho_b. \quad (17)$$

Pressure, which is the negative of the grand thermodynamic potential density, is given by

$$P = \sum_b \frac{1}{3} \frac{\gamma}{(2\pi)^3} \int d^3k \frac{k^2}{\sqrt{k^2 + m_b^{*2}}} (f_b + \bar{f}_b) + \frac{1}{2} m_\omega^2 \omega_0^2 - \frac{1}{2} m_\sigma^2 \sigma^2. \quad (18)$$

Mean-field value for the scalar meson σ is determined through the minimization of the thermodynamic potential, or equivalently maximizing the pressure with respect to the field.

Maximization of $P(m_b^*, \sigma)$ with respect to σ fields can be written as

$$\frac{dP}{d\sigma} = \sum_b \frac{\partial m_b^*}{\partial \sigma} \left(\frac{\partial P}{\partial m_b^*} \right)_{\mu_b, T} + \left(\frac{\partial P}{\partial \sigma} \right)_{m_b^*} = 0, \quad (19)$$

where

$$\left(\frac{\partial P}{\partial \sigma} \right)_{m_b^*} = -m_\sigma^2 \sigma, \quad (20)$$

and

$$\begin{aligned} \left(\frac{\partial P}{\partial m_b^*} \right)_{\mu_b, T} &= -\frac{\gamma}{3} \frac{1}{(2\pi)^3} \int d^3k \frac{k^2}{\epsilon_b^{*2}} \frac{m_b^*}{\epsilon_b^*} [f_b + \bar{f}_b] \\ &\quad - \frac{\gamma}{3} \frac{1}{(2\pi)^3} \frac{1}{T} \int d^3k \frac{k^2}{\epsilon_b^*} \frac{m_b^*}{\epsilon_b^*} [f_b(1 - f_b) + \bar{f}_b(1 - \bar{f}_b)] \\ &\quad - \frac{\gamma}{3} \frac{1}{(2\pi)^3} \frac{1}{T} g_{\omega b} \left(\frac{\partial \omega_0}{\partial m_b^*} \right)_{\mu_b, T} \int d^3k \frac{k^2}{\epsilon_b^*} [f_b(1 - f_b) - \bar{f}_b(1 - \bar{f}_b)] \\ &\quad + m_\omega^2 \omega_0 \left(\frac{\partial \omega_0}{\partial m_b^*} \right)_{\mu_b, T}. \end{aligned} \quad (21)$$

Since the baryon density ρ_b and temperature T are treated as input parameters, the variation of the vector mean field ω_0 with respect to the effective baryon mass m_b^* at a given value of the baryon density ρ_b in Eqs. (16) and (17) reads

$$\left(\frac{\partial \omega_0}{\partial m_b^*} \right)_{\mu_b, T} = - \frac{\frac{g_{\omega b}}{m_\omega^2} \frac{\gamma}{(2\pi)^3} \frac{1}{T} \int d^3k \frac{m_b^*}{\epsilon_b^*} [f_b(1 - f_b) - \bar{f}_b(1 - \bar{f}_b)]}{1 + \frac{g_{\omega b}^2}{m_\omega^2} \frac{\gamma}{(2\pi)^3} \frac{1}{T} \int d^3k [f_b(1 - f_b) + \bar{f}_b(1 - \bar{f}_b)]}. \quad (22)$$

Magnetic moments of baryons can be obtained by evaluating matrix elements of the magnetic moment operator as

$$\tilde{\mu}_b = \langle \Psi_b | \sum_{i=q} \hat{M}_i | \Psi_b \rangle, \quad (23)$$

where the sum is over the quarks in the bag, and $|\Psi_b\rangle$ is the wave function of a baryon b . The magnetic moment operator \hat{M}_i is given as

$$\hat{M}_i = \frac{\hat{Q}_i}{2} \mathbf{r}_i \times \boldsymbol{\alpha}, \quad (24)$$

where \hat{Q}_i and \mathbf{r}_i are the charge and the position operators of the i -th quark in the bag, and Dirac matrix $\boldsymbol{\alpha} = \gamma_0 \boldsymbol{\gamma}$. Details for the evaluation of the matrix elements can be found in Ref. [10], and we simply show the results for the analytic form for the magnetic moment of each baryon :

$$\begin{aligned} \mu_p(\rho, T) &= \frac{e}{2} D_u, & \mu_n(\rho, T) &= -\frac{e}{3} D_u, \\ \mu_\Lambda(\rho, T) &= -\frac{e}{6} D_s, & \mu_{\Sigma^+}(\rho, T) &= \frac{e}{6} \left[\frac{8}{3} D_u + \frac{1}{3} D_s \right], \\ \mu_{\Sigma^0}(\rho, T) &= \frac{e}{6} \left[\frac{2}{3} D_u + \frac{1}{3} D_s \right], & \mu_{\Sigma^-}(\rho, T) &= \frac{e}{6} \left[-\frac{4}{3} D_u + \frac{1}{3} D_s \right], \\ \mu_{\Xi^0}(\rho, T) &= -\frac{e}{3} \left[\frac{1}{3} D_u + \frac{2}{3} D_s \right], & \mu_{\Xi^-}(\rho, T) &= \frac{e}{6} \left[\frac{1}{3} D_u - \frac{4}{3} D_s \right]. \end{aligned} \quad (25)$$

where the integral D_q is defined as

$$D_q = \frac{4}{3} \mathcal{N}_q^2 \beta_q \left(\frac{R_b}{x_q} \right)^4 \int_0^{x_q} y^3 j_0(y) j_1(y) dy. \quad (26)$$

In the next section, we show numerical results and discuss them.

III. RESULTS AND DISCUSSION

First, we show the nucleon mass as a function of density and temperature in Fig. 1. At a temperature below 100 MeV, the nucleon mass decreases as density increases. Exchange of the σ meson mediates the scalar attraction. It leads to the reduction of the nucleon mass at finite baryon density. Since the effective mass of a baryon depends on σ meson fields in Eq. (11), the effective mass of the nucleon gets reduced. But the reduction is less in the QMC model than in the MQMC model, because the scalar-meson field in the QMC model is relatively smaller than the MQMC one.

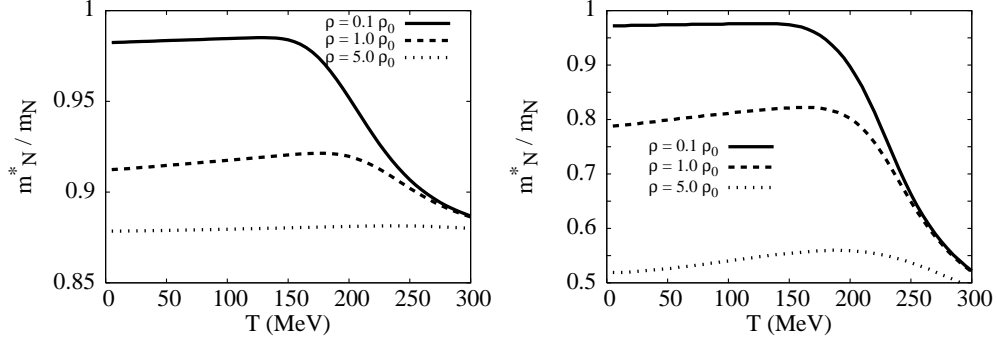


FIG. 1: Change of the nucleon mass with respect to density and temperature in the QMC (left) and the MQMC (right) models.

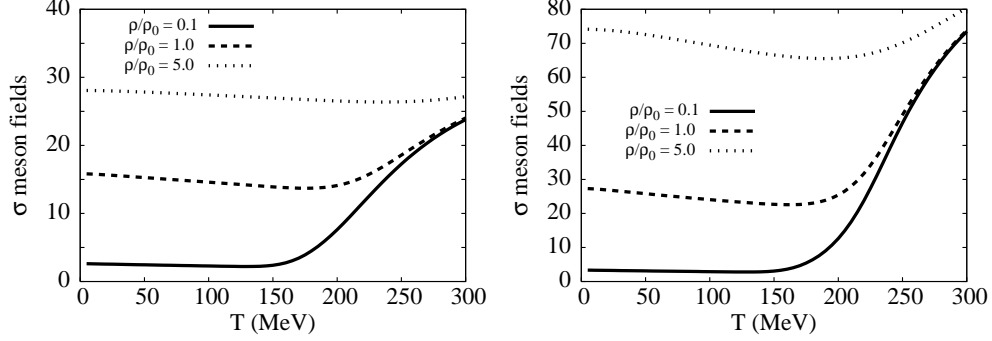


FIG. 2: Mean field of the σ meson at various temperatures and densities in the QMC(left) and the MQMC(right) models.

The effect of temperature becomes clearer as density is lower. At density $\rho/\rho_0 = 5$, temperature effect is almost invisible in the QMC model, and it is a minor correction compared to the density effect in the MQMC model. At $\rho/\rho_0 = 0.1$, change of the mass is mainly driven by temperature, starting at $T = 150$ MeV in both QMC and MQMC models. The density and temperature dependence of the nucleon mass in each model can be understood by looking at the behavior of the σ -meson field in Fig.2.

Fig. 2 shows the mean field values of σ -meson at various densities and temperatures. One can note that shapes of nucleon mass curves in Fig. 1 are highly correlated with the behavior of σ field in both models. For instance, in each model at $\rho/\rho_0 = 0.1$ and 1, the nucleon mass converges to a value as $T \rightarrow 300$ MeV. Similar convergence is observed from the σ field in both models. At small densities, thermal excitation of nucleons and anti-nucleons is the main source for the finite σ value, and it consequently leads to the decrease of the

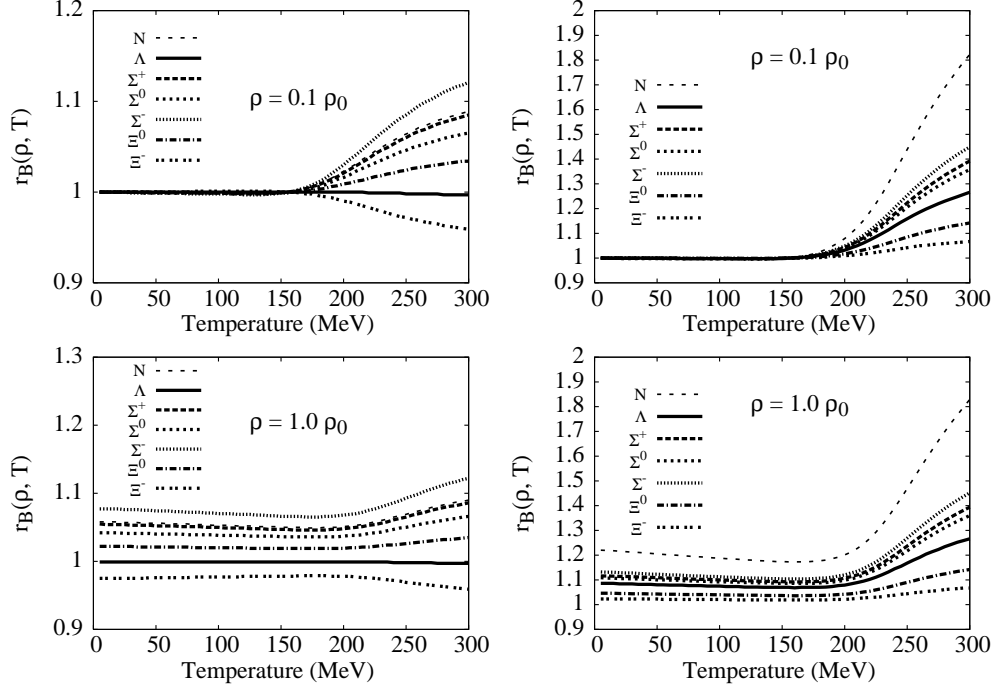


FIG. 3: Magnetic moments of octet baryons at $\rho/\rho_0 = 0.1$ (up row) and 1 (down row) in the QMC (left column) and the MQMC (right column) models. $r_b(\rho, T) \equiv \mu_b(\rho, T)/\mu_{b0}$ where μ_{b0} is the magnetic moment of a baryon b at $\rho = 0$ and $T = 0$.

nucleon mass. Since the σ field in the MQMC model builds up more rapidly than in the QMC model, mass reduction becomes drastic in the MQMC model.

Fig. 3 compares magnetic moments of octet baryons in the QMC and the MQMC model at $\rho/\rho_0 = 0.1$ and 1, where $r_b(\rho, T)$ is the ratio of the magnetic moment of a baryon b in medium of density ρ and temperature T relative to its free space value,

$$r_b(\rho, T) \equiv \frac{\mu_b(\rho, T)}{\mu_{b0}} \quad (27)$$

where μ_{b0} is the magnetic moment of a baryon b at $\rho = 0$ and $T = 0$. At density close to zero, temperature plays a dominant role in the change of the observable. In both models, the temperature effect begins at around $T = 150$ MeV, where non-zero σ field starts to have finite values, and the change of the magnetic moment becomes sizable at temperatures high enough.

Dependence on temperature is, however, contrastive in both models. Finite σ field causes the change of bag radius from its free space value by the minimal condition of the mass, Eq. (12). In Ref. [10], we studied the density effect to magnetic moments of octet baryons,

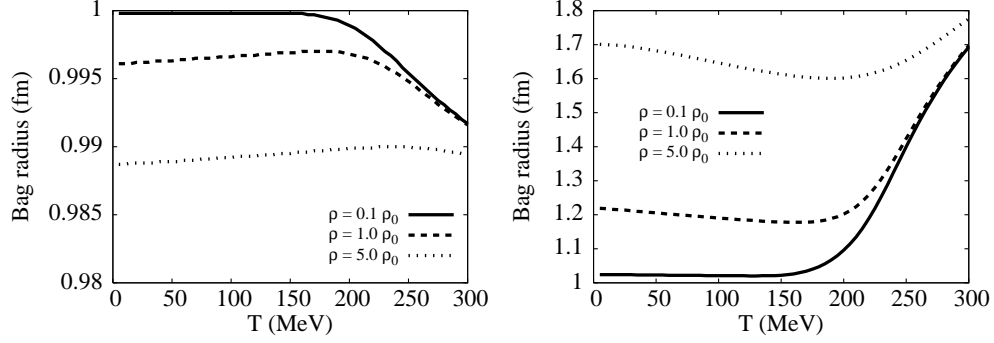


FIG. 4: Bag radius of the nucleon as a function of density and temperature in the QMC model (left) and the MQMC one (right).

and observed a close correlation between the bag radius and magnetic moments. Dependence on the temperature can be understood in a similar way as shown in Fig. 4.

In Fig. 4 we show the bag radius as a function of temperature at $\rho/\rho_0 = 0.1, 1.0$ and 5.0 . At zero temperature, the bag radius shrinks slightly from its free space value as density increases in the QMC model. Temperature effect shows a similar pattern, i.e. slightly decreases from free space radius as the σ field becomes finite at high temperatures. In the MQMC model, on the other hand, overall shapes of the curves for bag radius are very similar to those of the σ field, and they increase very quickly as temperature becomes high. As a result, change of the magnetic moment at finite temperatures in the MQMC model is more sensitive and significant than the QMC.

Another interesting result is the behavior of r_{Ξ^-} in Fig. 3, which decreases in the QMC model while it goes in the opposite direction in the MQMC one. Difference can be understood as follows. In Eq. (25), one can see formulae for the magnetic moments. Since $\mu_p > 0$, $\mu_\Lambda < 0$ and $\mu_{\Xi^-} < 0$ [10], it is obvious that $D_{u0} > 0$, $D_{s0} > 0$ and $D_{u0} - 4D_{s0} < 0$, where 0 in the subscript denotes the value in free space. In the QMC model, $r_N > 1$ and $r_\Lambda < 1$, which means that $\Delta D_u > 0$ and $\Delta D_s < 0$. Since $\Delta D_u - 4\Delta D_s > 0$ and $D_{u0} - 4D_{s0} < 0$, we have $\Delta r_{\Xi^-} = (\Delta D_u - 4\Delta D_s)/(D_{u0} - 4D_{s0}) < 0$ in the QMC model. On the other hand, $\Delta D_u > 0$ and $\Delta D_s > 0$ in the MQMC model, and thus the sign of Δr_{Ξ^-} cannot be easily determined as the QMC model. The numerical result indicates that indeed $\Delta D_u - 4\Delta D_s < 0$, and as a result $\Delta r_{\Xi^-} > 0$ in the MQMC model.

Magnetic fields in the relativistic heavy-ion collision have been recently considered, and

relevant theories predict a field strength as large as 1.5×10^{19} G for the LHC energy [11]. Magnetic fields of order 10^{18} G are also predicted in the core of the neutron star. With such high magnetic fields, baryon masses and the equation of state become substantially different from those without magnetic fields, and they subsequently affects the critical density for the transition to the deconfined quark matter and the maximum mass of the neutron star [12, 13].

Since a proton experiences the Landau quantization in the strong magnetic field, energies of a neutron and a proton may have different behaviors. However if we ignore the effect for a proton, the nucleon energy at densities close to zero can be estimated as

$$E_N \approx m_N - s\kappa_N B, \quad (28)$$

where B is the magnetic field, s is $+1$ (-1) for spin up (down) and κ_N is defined as $\kappa_b = (\mu_b/\mu_N - q_b m_p/m_b)\mu_N$ where μ_N is the magneton of a nucleon, $\mu_N = e/(2m_p) = 3.15 \times 10^{-18}$ MeV G $^{-1}$. In the free space, $m_N \simeq 940$ MeV and $\kappa_p \simeq 1.79\mu_N$ for a proton. Consequently, when $B/B_c^e \sim 10^5$ G ($B_c^e = 4.414 \times 10^{13}$ G is the critical electron field), the mass correction due to the magnetic field, $|\kappa_p B| \simeq 25$ MeV, is about 3 % of the free nucleon mass.

At sufficiently high temperature, the situation may be more drastic. We have shown in Figs. 1 and 3 that, as the temperature increases, the effective mass of the octet baryons decreases, but their magnetic moment increases from the free-space value. Consequently, total corrections to the nucleon mass by the strong magnetic fields at high temperature can be much larger than those in free space.

For instance, in the MQMC model, at $\rho/\rho_0 = 0.1$ and $T = 250$ MeV, effective mass of the nucleon gets reduced as about $0.65m_N$. Net effects due to temperature and magnetic fields give a correction of about 11 % of the effective mass of the nucleon, more than three times larger than that in free space. If we only take the free space value for the magnetic moment, then the correction due to the interaction with magnetic fields is about 6 %. Therefore, additional corrections due to strong magnetic fields may exhibit clear discrimination of finite temperature effects on the baryon mass and the magnetic moment.

IV. SUMMARY

We have considered the change of magnetic moments of octet baryons at a finite density and temperature. Two models, the quark-meson coupling model and its modified one, are employed to investigate any possible model dependence. Both models predict sizable effects due to the variation of density and temperature.

Drastic changes of magnetic moments with the increase of temperatures appear from about $T = 150 \text{ MeV}$ at a low density, while they change moderately at a high density. Consequently, if we understand the sudden changes of magnetic moments as a signal of the phase transition, our calculations may indicate the first order phase transition around $T = 150 \text{ MeV}$ at low densities and the second order phase transition at high densities. In addition, at high temperature, the effective mass of a nucleon for various densities is shown to converge to a value although hadronic models used here should be scrutinized at such high temperature.

As temperature increases, masses tend to decrease and magnetic moments increase, although there exists non-negligible model dependence on both models. If we consider strong magnetic fields which can be realized in the relativistic heavy ion collision and the neutron star, the change of magnetic moments and masses can give more significant corrections to the nucleon energy. Such changes of baryon properties at high temperature with the strong magnetic field may give an insight into the phase transition to the deconfinement and the restoration of broken symmetries. But, since strong magnetic fields cause the Landau quantization of charged particles and the breaking of spherical symmetry, more detail calculations are to be done. Investigation along this direction will be considered in near future.

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